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Theoretical Estimation of Fracture Toughness of Fibrous Composites

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A method of estimating the fracture surface energy of fibre-reinforced materials is discussed. The surface work is shown to increase with increasing fibre content, strength and diameter, and decrease with increasing fibre modulus and matrix flow stress (or hardness).

Relatively short fibres should be used if high toughness is required, and the maximum toughness that can be achieved is limited by the amount of crack opening that can be permitted. Under certain conditions, incorporation of fibres into a material can lead to embrittlement.

1. Introduction

Fibre-strengthening has so far mainly been considered for the strengthening of materials which are weak because of having a low flow stress. The incorporation of strong fibres has been shown theoretically [1, 2] and practically [3] to lead to considerable increases in the strength of plastics and ductile metals.

Fibre-strengthening can under favourable circumstances produce composites which are tough, the toughness being due to delamination of the material parallel to the axis of the applied stress [4, 5]. In addition a long stress transfer length favours toughness [6].

The toughening of glass by using stressed fibres has been tested experimentally with good results [7]. Also a method of fibre-toughening has been suggested [8] in which ductile, tough fibres are used to arrest crack development. In this however, no account was taken of the effect on fibre toughness of the restraint provided by the matrix. The matrix will restrict plastic flow in the fibres as long as they are well bonded to it, and this could drastically reduce fibre toughness, and thus reduce their toughening effect on the matrix.

Composites with parallel fibres stressed along the fibre direction fail in a manner that appears to depend on the bond between fibre and matrix. In the case of silica fibre-reinforced aluminium, when the bond was poor, cracks in the matrix normal to the stress axis were deflected by the fibres, and failure occurred by delamination. When bonding was good, however, failure occurred by the crack extending normal to the stress axis and breaking the fibres [9]. During this process bridging of the crack by fibres may have been occurring; such bridging has been observed in the case of glass fibres in epoxy resins [10].

When fibre-bridging takes place, the stress on the fibres should be maximum in the plane of the crack. Thus, while short fibres may pull out, or fibres with many points of weakness in them may fail some distance from the crack and pull out [11], uniformly strong fibres can be expected to fracture in the crack plane. Before they fracture they may contribute to the fracture surface energy of the composite, if the matrix is ductile, by causing plastic flow in the matrix at the fibre surface. When they fracture the elastic energy in them may not be recoverable because they could cause further plastic flow in the matrix as they relax.

This type of process is examined quantitatively, and an expression for fracture toughness is developed, ignoring elastic stress transfer. Then elastic stress between fibres and matrix is also considered. Finally some practical implications are discussed.

2. Theory

2.1. Model Considered

An infinite sheet of composite will be considered, containing parallel, continuous fibres. The adhesive bond between fibres and matrix is assumed not to fail. The composite contains a crack normal to the fibres, and the work required to extend the crack will be discussed.

The matrix, which in the absence of fibres, would be isotropic, with rigidity modulus G, is elastic in shear up to stress τ_y. At shear stress greater than τ_y it flows with no further increase in stress. The fibres, with Youngs Modulus E_f, are assumed uniform in properties, and are elastic in tension up to stress σ_u, at which point they break without further deformation.

When the crack extends, the matrix is assumed to fail, doing surface work γ_m/unit area of crack which is substantially the same as that required to cause crack advance in the absence of fibres. The mechanism of matrix failure is assumed to involve some process which does not significantly alter the properties of the matrix close to the crack face. Stress release in the matrix on either side of the failed region is assumed to occur, and fibres are assumed initially to bridge this region. But as the crack opens, due to the applied stress σ_c, the fibres will be stretched, re-stressing the matrix to some extent. With sufficient opening of the crack, the fibres will reach the failure strain, σ_u/E_f. Stress transfer between fibres and matrix takes place by the combined mechanisms discussed by Piggott [2].

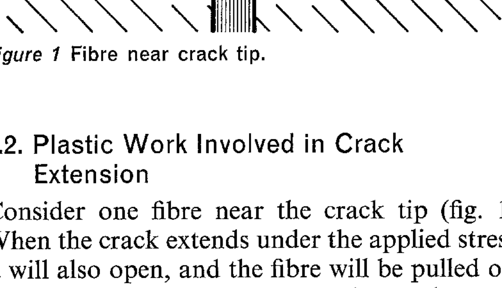

Figure 1 Fibre near crack tip.

2.2. Plastic Work Involved in Crack Extension

Consider one fibre near the crack tip (fig. 1). When the crack extends under the applied stress, it will also open, and the fibre will be pulled out of the matrix to some extent. Owing to the stress transfer between fibre and matrix, the fibre stress, and hence strain, will vary along the fibre length from a maximum value at the crack face, to the average fibre stress at some region remote from the crack. The fibre will do work on the matrix, which, with the assumptions discussed, is given at any point by the product of the interfacial force and the displacement of the fibre with respect to the matrix.

Let the displacement be u and the work done be U_m, then since the force transmitted from fibre to matrix over a length dx of fibre is $\pi d\tau_y u dx$, the work done by the fibre on the matrix is

$$U_m = \pi d \int_0^{x_0} \tau_y u \, dx \tag{1}$$

neglecting elastic stress transfer. x_0 is the distance along the fibre to the point where the fibre stress has fallen to the average value.

For the present we will neglect the average fibre stress.

U_m will increase until the fibre breaks, and will not be available to do work on other parts of the system when fibre failure occurs. The fibre will also gain elastic energy as fibre stress increases, and when the fibre breaks most of this energy will be released by the fibre doing further work on the matrix. This energy, which will also not be available for other work, will be given by

$$U_f = \tfrac{1}{2} \int_0^{x_0} P \epsilon_f \, dx \tag{2}$$

where P is the force transmitted along the fibre and ϵ_f is the fibre strain. Each fibre intersected by the crack will thus contribute an amount of energy $U_m + U_f$ to the fracture surface energy. U_m and U_f now refer to the values of the energies when the fibre is at the breaking point, and may be evaluated as follows.

Stress transfer from fibre to matrix will result in the fibre force decreasing with distance from the crack surface; i.e.

$$\frac{dP}{dx} = -\pi d \tau_y \, .$$

If the fibre is restrained from decreasing in diameter, the strain in the fibre is given by

$$\epsilon_f = \frac{4P(1-\nu_f)(1-2\nu_f)}{\pi d^2 E_f (1-\nu_f)} = \frac{4P}{\pi d^2 E_{fr}}$$

say, where

$$E_{fr} = \frac{(1-\nu_f)E_f}{(1+\nu_f)(1-2\nu_f)} \, .$$

Thus

$$\frac{d\epsilon_f}{dx} = -\frac{4\tau_y}{dE_{fr}} \tag{3}$$

and by integration with the boundary condition $\epsilon_f = \sigma_u / E_{fr}$ when $x = 0$, the stress transfer length x_0 is given by

$$x_0 = \frac{d\sigma_u}{4\tau_y} \quad (4)$$

and for $x < x_0$

$$\epsilon_f = \frac{4\tau_y}{dE_{fr}}(x_0 - x) \quad (5)$$

since τ_y is assumed constant. Thus

$$u = \int_{x_0}^{x} \epsilon_f dx \cdot \quad (6)$$

Substituting values of x_0, ϵ_f and u from equations 4 to 6 into 1 and 2 and integrating gives

$$U_m = U_f = \frac{\pi d^3 \sigma_u^3}{96\tau_y E_{fr}} \cdot \quad (7)$$

Thus n fibres/unit area contribute a fracture surface energy

$$\gamma_f = n(U_m + U_f) = \frac{\pi n d^3 \sigma_u^3}{48\tau_y E_{fr}} = \frac{p d \sigma_u^3}{12\tau_y E_{fr}} \quad (8)$$

where $p = n\pi d^2/4$ is the proportion of fibres in the composite.

Fracture toughness, $K = \sigma_c\sqrt{\pi c}$ where σ_c is the applied stress for failure, and $2c$ is the crack length, may be calculated from the Griffith equation

$$\sigma_c = \sqrt{\frac{2E_m\gamma_m}{\pi(1 - \nu^2_m)c}}$$

for a thin crack in an infinite sheet in tension normal to the crack. (E_m = Young's modulus, γ_m = fracture surface energy and ν_m = Poisson's ratio of the matrix.) Thus, the fracture toughness of the matrix alone is given by

$$K_m = \sqrt{\frac{2E_m\gamma_m}{(1 - \nu^2_m)}}$$

and the fracture toughness of the composite is

$$K_c = \sqrt{\frac{2E_c}{(1 - \nu^2_c)}\left\{(1 - p)\gamma_m + \frac{p d \sigma_u^3}{12\tau_y E_{fr}}\right\}} \quad (9)$$

where E_c is the modulus and ν_c is Poisson's ratio of the composite.

3. Discussion

Equations 8 and 9 enable values for the fracture surface energy and toughness of composites to be calculated only under conditions where the elastic stress transfer near cracks for fibres bridging the cracks is small compared with stresses transferred by plastic flow at the interface between matrix and fibres. In addition, for simplicity in deriving the equations the fibres were assumed to be unstressed in regions remote from the crack. These stresses will now be considered, and the case of stress transfer by friction between fibres and matrix will be discussed. The implications for practical materials will also be examined.

3.1. Effect of Elastic Stress Transfer

The fibre stress arising from elastic stress transfer between fibres and matrix, for fibres which are very long is

$$\sigma_e = \tau_y \sqrt{\frac{2E_{fr}}{G}\log(3.63/p)}$$

(from equation A5, Appendix 1) where G is the modulus of rigidity of the matrix. The relative importance of this fibre stress is thus

$$\frac{\sigma_e}{\sigma_u} = \frac{\tau_y}{\sigma_u}\sqrt{\frac{2E_{fr}}{G}\log(3.63/p)} \cdot \quad (10)$$

The energy in the fibre resulting from elastic stress transfer, given by equation A6 yields

$$\frac{U_{fe}}{U_f} = \frac{12E_{fr}^2\tau_y^2\log(3.63/p)}{G\sigma_u^3} \quad (11)$$

and the extra elastic energy in the matrix due to stress transfer near the crack tip is given by

$$U_{me} = U_{fe}\left\{\frac{\sigma_u}{E_{fr}} + \frac{\tau_y^2}{2E_{fr}G}\log(3.63/p)\right\}$$

(equation A7). For strong fibres σ_u/E_{fr}, the breaking strain of the fibres, is not likely to exceed 0.1, and is often in the region of 0.01. Thus, for a matrix whose yield stress is not very high U_{me} can be neglected compared with U_{fe}. The ratio U_{fe}/U_f may therefore be used to evaluate the toughening or embrittling effect resulting from the presence of the fibres.

If $U_{fe}/U_f = 2$, the fibres introduce equal amounts of elastic and plastic work near the crack tip. Thus, they can be expected to have little effect on fracture toughness. However, when $U_{fe}/U_f < 2$, the fibres will toughen the matrix, and when $U_{fe}/U_f > 2$ the fibres can be expected to cause embrittlement.

$U_{fe}/U_f = 2$ will occur when the elastic contribution to fibre stress is relatively low, if the fibre breaking strain is low. For example, for $\sigma_u/E_{fr} = 0.01$, $U_{fe}/U_f = 2$ when $\sigma_e = 0.058\sigma_u$. Low values of U_{fe}/U_f are favoured by fibres that break at high strain and high stress, and matrices that flow when the stress and strain are small. In addition, fibre content should be as large as possible. The values of γ_f and K_c given by equations 8 and 9 are only valid when $U_{fe}/U_f \ll 2$.

Work-hardening in the matrix will increase the recoverable elastic energy, and decrease the toughening effect. Consequently the toughness calculated from these formulae will be too high for work-hardening matrices.

3.2. Effect of Substantial Average Fibre Stress

In regions remote from a crack the fibres will be under stress from the normal fibre-strengthening stress transfer (given by equations 1 and 6 in [2], for example). By considering the maximum fibre stress due to this form of stress transfer, σ_s say, we may calculate the maximum effect we can expect this to have.
(For long fibres $\sigma_s = E_f\ \sigma_c/pE_f + (1 - p)E_m$.) When $x = x_0$ the fibre stress is assumed to be σ_s instead of zero. Thus, x_0 (equation 4) is reduced to

$$x_0 = \frac{d(\sigma_u - \sigma_s)}{4\tau_y}\ . \qquad (4a)$$

If the integrations in equations 1, 2 and 5 are done for this value of x_0, this gives for equation 8

$$U_m = U_f = \frac{\pi d^3(\sigma_u{}^3 - \sigma_s{}^3)}{96\tau_y E_{fr}}\ . \qquad (8a)$$

This indicates that normal fibre-strengthening stress transfer is not important unless the applied stress is so high that the average fibre stress reaches a significant fraction of the ultimate fibre strength.

3.3. Stress Transfer by Friction

If in making the composite the matrix has been shrunk onto the fibres, so that there is a radial compressive stress at the fibre surface and no adhesive bond, stress may be transferred by friction between matrix and fibres. For constant coefficient of friction the interfacial axial shear stress will vary due to shrinkage of the fibre diameter arising from Poisson's ratio. Thus, in equation 4, τ_y should be replaced approximately by $\mu(R - E_{mr}\epsilon_f)$ where R is the interfacial radial stress for fibres that are not axially stressed, and $E_{mr}\epsilon_f$ is an approximate estimation of the reduction in interfacial stress due to "Poisson's Ratio" effect (see Appendix 2). So long as this fibre shrinkage is not too large, i.e. when $\sigma_u E_{mr}/E_{fr}R \ll 1$, x_0, U_m and U_f are given by equations 4 and 7 with $\tau_y = \mu R$. Some correction is needed for larger shrinkage factors, and for $\sigma_u E_{mr}/E_{fr}R \gtrsim 1$ the values of tensile strain and Poisson's ratio in the matrix have to be taken into account.

3.4. Discontinuous Fibres

Having fibres of relatively low aspect ratio would reduce the amount of elastic stress transfer, and hence increase toughness, in cases where $U_{fe} \gtrsim U_f$. The fibres have to have lengths greater than $2x_0$ to prevent them being pulled out. However, even if the fibre length is $4x_0$ at least half the fibres will pull out, though in doing so they will still contribute something to fracture toughness. For any composite there will clearly be an optimum fibre length for maximum fracture toughness.

If stress is transferred by friction rather than flow in the matrix, the formulae given here can only be used to calculate the optimum fibre length when $\sigma_u E_{mr}/E_{fr}R < 1$.

3.5. Practical Implications of Parameters Affecting Toughness

Consideration of elastic as well as plastic stress transfer (section 3.1) indicates that embrittlement may result from the addition of fibres to a ductile matrix. Equation 11 suggests that this will occur when long, high modulus fibres are added to a matrix with low modulus, especially if the fibres are not particularly strong. If carbon fibres, modulus 4×10^4 kg mm^{-2} and strength 700 kg mm^{-2} are added to a plastic, for example, with $\tau_y{}^2/G = 1/10$ kg mm^{-2} we find that $U_{fe}/U_f \simeq 12$, and the composite can be expected to be brittle. The embrittling effect can be reduced by having a coating on the fibres which has a much lower flow stress than the matrix. τ_y and G in the equations developed here may then be replaced by the flow stress and shear modulus of the coating. Alternatively, as mentioned in section 3.4 the fibre length should not be very much greater than $2x_0$.

So long as $U_{fe}/U_f \ll 2$, it should be possible to produce a tough material. However, the toughness that may be achieved is limited by the maximum permissible crack opening. The full toughening effect of the fibre is produced when the crack has opened by an amount $2u_0$ in the region of the fibre, where u_0 may be obtained by integrating equation 6 between x_0 and zero; thus,

$$2u_0 = \frac{d\sigma_u^2}{4\tau_y E_{fr}} \cdot$$

γ_f may be maximised and u_0 minimised if the ratio $\gamma_f/2u_0 = p\sigma_u/3$ is as large as possible. Consequently, the useful toughening that may be obtained is limited, and is greatest when strong fibres are used in as high a proportion as possible.

4. Conclusions

By choice of suitable values for fibre strength, diameter and modulus, together with a matrix (or fibre coating) with low flow stress, very tough composites can be made. With unsuitable combinations of materials however, brittle composites can be produced. Brittleness can occur even when elastic stress transfer to the fibres is small. This difficulty can be eliminated, however, by using short fibres instead of long ones, but full toughening will not be achieved if too many of the fibres pull out of the matrix instead of breaking. The toughness estimated from the equations will be too high for matrices that work-harden.

Stress transfer by friction between fibres and matrix can give very tough composites, but calculation of the critical stress transfer length, and hence optimum fibre length, is complicated by fibre shrinkage.

The amount of toughening that may be achieved is limited by the maximum permissible crack width in a composite which does not fail. Pre-requisites for high toughness are high values of fibre strength and fibre content.

Appendix 1

Elastic Stress Transfer

The treatment in [2] for elastic stress transfer may be adapted to calculate the stress and energy in the fibre where the matrix is still elastic. Let the shear modulus of the matrix be G. If u_m is the displacement in the matrix parallel to the fibre at the fibre surface, the matrix strain in this region is $2u_m/d \log D/d$, where D = average distance between nearest neighbour fibres, so that the stress at the matrix-fibre interface is

$$\tau = 2u_m G/d \log D/d$$

and assuming no slip between fibre and matrix, the force in the fibre is

$$P = -\tfrac{1}{4}\pi d^2 E_f \frac{du}{dx}$$

(u increases as x decreases with axis chosen as in fig. 1). Since

$$\frac{dP}{dx} = -\pi d\tau \qquad (A1)$$

$$\frac{d^2P}{dx^2} = \frac{8GP}{d^2 E_{fr} \log D/d}$$

which for $P = P_0$ at $x = x_0$ and $P = 0$ at $x = L$ (L = fibre length on each side of the crack) has the solution

$$P = P_0 \frac{\sinh \beta(L - x)}{\sinh \beta(L - x_0)} \qquad (A2)$$

where

$$\beta^2 = \frac{8G}{d^2 E_{fr} \log D/d} = \frac{8G}{d^2 E_{fr} \log(2\pi/p\sqrt{3})} \qquad (A3)$$

where p is the proportion of fibres in the composite, assumed packed in hexagonal array (see equation 4, [2]). τ may be calculated by substituting the value of P from equation A2 into equation A1, i.e.

$$\tau = \frac{\beta P_0}{\pi d} \frac{\cosh \beta(L - x)}{\sinh \beta(L - x_0)} \cdot \qquad (A4)$$

However, at $x = x_0$, $\tau = \tau_y$ since this is where the matrix becomes plastic at the fibre-matrix interface; thus

$$\tau_y = \frac{\beta P_0}{\pi d} \coth \beta(L - x_0) \cdot$$

For very long fibres βL is large, so that

$$\sigma_s = \frac{4P_0}{\pi d^2} = \tau_y \sqrt{\frac{2E_{fr}}{G} \log(2\pi/p\sqrt{3})} \cdot \qquad (A5)$$

The energy in the fibre due to elastic stress transfer is

$$U_{fe} = \int_{x_0}^{L} P dx \,,$$

which using equation (A2) for P, gives

$$U_{fe} = \frac{\pi d^3 \tau_y E_{fr}}{8G} \log(2\pi/p\sqrt{3}) \qquad (A6)$$

when L is large.

The energy in the matrix due to elastic effects is

$$U_{me} = \frac{\pi}{2} d^2 \log D/d \int_0^L \frac{\tau^2}{2G} dx =$$

$$\frac{\pi}{2} d^2 \log(2\pi/p\sqrt{3}) \left\{ \int_0^{x_0} \frac{\tau_y^2}{2G} dx + \int_{x^0}^{L} \frac{\tau^2}{2G} dx \right\},$$

which using equation A4 for τ, gives

$$U_{me} = \left\{\tau_y \sqrt{\frac{E_{fr}\log(2\pi/p\sqrt{3})}{2G}} + \sigma_u\right\}\frac{\pi d^3\tau_y}{16G}\log(2\pi/p\sqrt{3})$$

$$= \left\{\tau_y \sqrt{\frac{\log(2\pi/p\sqrt{3})}{2GE_{fr}}} + \frac{\sigma_u}{E_{fr}}\right\} U_{fe} \cdot \quad \text{(A7)}$$

Appendix 2

Shear Stress not Constant

The importance of stresses which can develop normal to the fibre surface in stressed fibrous composites due to differences in the mechanical properties of the fibres and matrix has been demonstrated by Ebert and co-workers (see e.g. [13]). A significant effect of these stresses is that in the absence of an adhesive bond, stress transfer can take place by frictional forces between matrix and fibre. In the region close to a crack face the stresses in the matrix are assumed (in the model discussed here) to fall to a low value. Consequently, the radial stresses at the fibre surface will be small, and stress transfer to the matrix will be reduced in these regions. The interfacial shear stress will therefore not be constant, as assumed in the integration of equation 3 above.

This situation can be treated approximately by letting the radial stress at the fibre matrix interface in regions remote from the crack face have the value R. Close to the crack face, fibres subject to a strain ϵ_f will be compressed by a radial stress of approximately

$$R - \frac{\nu_f E_m}{1 + \nu_m}\epsilon_f \cdot$$

Let $E_{mr} = \nu_f E_m/(1 + \nu_m)$. Then in equation 3, τ_y should be replaced by $\mu(R - E_{mr}\epsilon_f)$ where $\mu =$ coefficient of friction between fibres and matrix i.e.

$$\frac{d\epsilon_f}{dx} = \frac{4\mu(R - E_{mr}\epsilon_f)}{dE_{fr}}$$

and following the steps outlined in the development of the theory we find, writing

$$\phi = \frac{\sigma_u E_{mr}}{E_{fr}R} \quad \text{(A8)}$$

that so long as $\phi < 1$, the fibres will break in the region of the crack in the same way as for the theory presented above, and the critical fibre length and fibre and matrix energies are

$$x_0' = \frac{dE_{fr}}{4\mu E_{mr}}\log\left(\frac{1}{1-\phi}\right) \quad \text{(A9)}$$

$$U_f' = \frac{\pi\sigma_u^3 d^3}{32\mu R E_{fr}} \left\{-\frac{1}{\phi^3}\left(\log(1-\phi) + \phi\left(1 + \frac{\phi}{2}\right)\right)\right\} \quad \text{(A10)}$$

$$U_m' = \frac{\pi\sigma_u^3 d^3}{16\mu R E_{fr}} \left\{\frac{1-\phi}{\phi^3}\log(1-\phi) + \frac{2-\phi}{2\phi^2}\right\}. \quad \text{(A11)}$$

Finally writing $\tau_y = \mu R$, for small ϕ these equations reduce to

$$x_0' = x_0\left(1 + \frac{\phi}{2}\right) \quad \text{(A12)}$$

$$U_f' = U_f\left(1 + \frac{3\phi}{4}\right) \quad \text{(A13)}$$

$$U_m' = U_c\left(1 + \frac{\phi}{2}\right) \cdot \quad \text{(A14)}$$

For $\phi \simeq 1$ or $\phi > 1$ the approximation associated with letting the radial stress be $R - E_{mr}\epsilon_f$ is inadequate.

Symbols Used

c = half length of crack.
d = fibre diameter.
D = average separation of nearest neighbour fibres.
E = Young's modulus.
G = shear modulus of matrix.
K = fracture toughness.
L = length of fibre on either side of crack.
n = number of fibres per unit area.
P = force on fibre.
R = pressure exerted by matrix on fibres.
u = displacement.
U = work.
x = distance.

Greek Symbols

$\beta = 8G/d^2E_{fr}\log(2\pi/p\sqrt{3})$.
γ = surface energy or work.
ϵ = tensile strain.
$\phi = \sigma_{fm}E_{mr}/E_{fr}R$.
σ = tensile stress.
τ = shear stress.
μ = coefficient of friction.
ν = Poisson's ratio.

Suffixes

c = composite.
e = elastic.
f = fibre.
m = matrix.

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